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Th. 5 If $V(F)$ be a F.G.V.S. Then foll. statements are equivalent.

1. $S \subseteq V$ is L.I. and $\langle S \rangle = V$
2. S is a basis of V
3. S is a MAXIMAL L.I.
4. S is MINIMAL SUBSET of generators of V.
5. Every elt. of V can be written as the linear combination of the elts. of S.

Eg: $V(\mathbb{R}) = P_2(\mathbb{R})$
$S = \{1, x, x^2\} \rightarrow \boxed{BASIS}$
$\langle S \rangle = V$
no proper subset of S generates $V \Rightarrow S$ is mini. subset of generator

Let $S = \{1, x, x^2, a_0 + a_1 x + a_2 x^2\} \rightarrow$ L.D.
No superset of S is L.I.
$\Rightarrow S$ is $\boxed{\text{maximal}}$ L.I. set.

Quest: Which of the foll. V.S. has finite basis?
(A) $\mathbb{R}(\mathbb{Q})$ ×   (B) $\mathbb{C}(\mathbb{Q})$ ×   (C) $\mathbb{R}^2(\mathbb{R})$ ✓   (D) $\mathbb{Q}(\mathbb{Q})$ ✓
$B = \{(1,0), (0,1)\}$
✓ (E) $\{0\}$